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Translated by M.D.F.

PMM U.S.S.R.,Vol.50,No.5,pp.658-662,1986
Printed in Great Britain

0021-8928/86 $10.00+0.00

THE ASYMPTOTIC STABILITY OF SYSTEMS WITH DELAY*

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Sufficient conditions for the existence of a finite domain of attraction of an unperturbed solution of autonomous system with delay are obtained, and its lower estimate is given using a method which requires that only the Lyapunov function need be known for the system in question without delay.

The results of investigating the stability of non-linear systems with delay /1, 2/ enable one to determine the domain of stability in the parameter space of the non-linear problem and of the domain of attraction of the unperturbed solution for mainly autonomous and non-autonomous first-order systems.

Following /3/, the application of the Lyapunov vector function is proposed in order to use the methods described in /1, 2/ constructively for systems of higher order.

Let the unperturbed motion $x=0$ of the system

$$x_i^{\cdot} = f_i(x(t)) + \sum_{j=1}^{m} F_{ij}(x(t))\, u_j(x(t-\tau)), \quad \tau = \text{const} \geqslant 0 \tag{1}$$

$$x \in R^n, \quad u \in R^m, \quad f_i,\ F_{ij},\ u_j \in C^1(\Omega), \quad \Omega \subset R^n, \quad m \leqslant n$$

without delay ($\tau=0$) by asymptotically stable, and let a Lyapunov function $V(x)$, positive definite in the convex region $\Omega_0 \subset \Omega$ be known for (1), with the time dertivative of this function negative definite in Ω by virtue of the system (1) ($\tau=0$). Some or all (when $m=n$) functions $f_i(x(t))$ here can be identically equal to zero.

We take $t=0$ as the initial instant. Let the initial continuous curve be described, at $\tau \leqslant t \leqslant 0$, by the function $\Phi(t) \in \Omega$. We shall write it in the form of a sum $\Phi(t) = \varphi(t) + \psi(t)$ where $\|\varphi(t)\| \leqslant \varphi^*$, $\varphi^* = \text{const}$ ($\|\cdot\|$ is the Euclidean norm of a vector) and the function $\psi(t)$, $\psi(0) = x(0)$ is a solution of system (1) for $\tau=0$. We will assume, without loss of generality, that $\varphi(0)=0$ and call the function $\psi(t)$ the reference function.

Let the domain Ω^* together with its boundary $\partial\Omega^*$ defined by the equation $V(x) = v^*$, $v^* = \text{const} > 0$ lie within Ω_0. Such a domain will be the domain of attraction of the unperturbed solution $x=0$ of system (1) at $\tau=0$ /1/. The domain may collapse when $\tau \neq 0$. Below we shall consider the bounded domains only. If on the other hand the motion $x=0$ is asymptotically stable in the large when $\tau=0$, then the bounded domain Ω_0 can be chosen arbitrarily.

We shall regard as the domain of attraction of the unperturbed motion $x=0$ of system (1), the sets of points of the phase space representing the initial values of the solutions of (1) tending, as $t \to \infty$, to the unperturbed motion $x=0$ for any initial functions $\Phi(t)$ belonging to the class specified above.

Let us clarify the conditions imposed on the parameters τ and φ^* under which the region Ω^* remains within the domain of attraction of the unperturbed motion.

Let us write the solution of system (1) ($\tau \neq 0$) within the time interval $\tau \leqslant t \leqslant 0$ in the form of a sum $x(t) = \psi(t) + y(t)$ where $y(t) = \varphi(t)$ and $t \in [-\tau, 0]$. Then, when $t \geqslant 0$, the function $y(t)$ will, according to (1), satisfy the differential vector equation which can be written in the form

$$y^{\cdot} = \sum_{k=1}^{4} F_k(y, \psi, \tau) \tag{2}$$

$$F_1 = f(\psi+y) + F(\psi+y)\, u(\psi+y) - f(\psi) - F(\psi)\, u(\psi)$$

$$F_2 = F(\psi+y)\, [u(\psi) - u(\psi+y)], \quad F_3 = F(\psi+y)\, \{u[\psi(t-\tau) + y(t-\tau)] - u[\psi(t-\tau)]\}, \quad F_4 = F(\psi+y)\, \{u[\psi(t-\tau)] - u[\psi(t)]\}$$

**Prikl.Matem.Mekhan.*,50,5,851-855,1986

(Here and sometimes below the argument t will not be written explicitly in functions independent of τ).

The solution $x(t)$ of system (1) passing at $t=0$ through the point $x_0 \in \bar{\Omega}^*$ ($\bar{\Omega}$, is the closure of the domain Ω). Irrespective of the initial function $(\varphi(t)+\psi(t))$, $\psi(0)=x_0$, it cannot leave the domain Ω_0 in a time shorter than

$$t^* = \frac{a^*}{A^*}, \quad A^* = \max_{x,\, z \in \bar{\Omega}_0} \left\{ \sum_{i=1}^{n} \left[f_i(x) + \sum_{j=1}^{m} F_{ij}(x)\, u(z) \right]^2 \right\}^{1/2}$$

where a^* is the distance between the surfaces $\partial\Omega^*$ and $\partial\Omega_0$ ($\partial\Omega_0$ is the boundary of Ω_0 defined by the equation $V(x)=v_0$).

Therefore, if the region Ω is not the same as R^n, we will adopt a preliminary limitation for the delay: $\tau \leqslant t^*$. The solution $x(t), 0 \leqslant t \leqslant \tau$ will have the corresponding solution $y(t), 0 \leqslant t \leqslant \tau$ of system (2) with initial function $\varphi(t)$.

Let us estimate, over the norm, the solution $y(t)$ within the time interval $[0, \tau]$. The region Ω_0 is convex, therefore we can give the following upper estimate /4/ for the norms of the terms appearing on the right-hand side of (2):

$$\begin{gathered} \|F_1\| \leqslant K_1\|y\|, \quad \|F_2\| \leqslant K_2K_3\|y\| \\ \|F_3\| \leqslant K_2K_3\|y(t-\tau)\|, \quad \|F_4\| \leqslant K_2K_3\|\psi(t-\tau)-\psi(t)\| \\ K_1 = n^{3/2}a_1, \quad a_1 \geqslant \left|\frac{\partial h_i(x)}{\partial x_k}\right|, \quad h_i(x) = f_i(x) + \sum_{j=1}^{m} F_{ij}(x)\, u_j(x) \\ K_2 = n\sqrt{m}\; a_2, \quad a_2 \geqslant \left|\frac{\partial u_j(x)}{\partial x_k}\right|, \quad K_3 \geqslant \left[\sum_{i=1}^{n}\sum_{j=1}^{m} F_{ij}^2(x)\right]^{1/2} \end{gathered} \tag{3}$$

$x \in \bar{\Omega}_0$; $i, k = 1, \ldots, n$; $j = 1, \ldots, m$.

Here $\psi(t)$ is the reference solution corresponding to the solution $x(t)$, i.e. $\psi(0) = x(0)$. Since

$$|\psi_i(t-\tau) - \psi_i(t)| = \left|\int_{t-\tau}^{t} h_i(x(s))\, ds\right|$$

we have

$$\|\psi(t-\tau) - \psi(t)\| \leqslant A\tau, \quad A = \max\left\{\sum_{i=1}^{n} h_i^2(x)\right\}^{1/2}, \quad x \in \bar{\Omega}_0$$

Therefore we have the following inequality for system (2):

$$\|y^{\cdot}\| \leqslant a\|y\| + b\,\|y(t-\tau)\| + bA\tau, \quad a = K_1 + b, \quad b = K_2K_3 \tag{4}$$

Using the Cauchy inequality we obtain $\|y\|^{\cdot} \leqslant \|y^{\cdot}\|$. Then, introducing the new variable $z = \|y\|$ and remembering that $z(t-\tau) = \|\varphi(t-\tau)\|$, we can write for the system (2) when $0 \leqslant t \leqslant \tau$, the following differential scalar inequality:

$$z^{\cdot} \leqslant az(t) + b\varphi_0(t) + bA\tau, \quad \varphi_0(t) = \|\varphi(t-\tau)\| \tag{5}$$

Taking into account the fact that $z(0) = 0$ and employing the method used to prove assertion D in /5, p.172/, we obtain

$$z(t) \leqslant c(A\tau + \varphi^*)(e^{at} - 1), \quad c = b/a \tag{6}$$

Thus the deviation in the norm of the trajectory of system (1) from the reference trajectory in the interval $0 \leqslant t \leqslant \tau$, does not exceed a quantity $z(\tau)$ estimated by the right-hand side of inequality (6).

Let us denote by $\{\psi^k(t)\}$ $(k = 1, 2 \ldots)$ the sequence of reference trajectories for the solution $x(t)$ (1), satisfying the conditions $\psi^k(k\tau) = x(k\tau)$. We have the following estimate for the deviation of $x(t)$ from $\psi^1(t), t \in [0, \tau]$:

$$\begin{gathered} \|\psi^1(t) - x(t)\| \leqslant \|\psi^1(t) - \psi(t)\| + \|\psi(t) - x(t)\| \leqslant \\ \|\psi^1(\tau) - \psi(\tau)\|\, e^{na_1\tau} + z(\tau) \leqslant z(\tau)(1 + e^{na_1\tau}) \end{gathered} \tag{7}$$

Note that inequality (7) also holds for any reference solution $\psi^{\lambda}(t)$, $\psi(\lambda\tau) = x(\lambda\tau)$, $\lambda \in [0,1]$ when $t \in [-\lambda\tau, (1-\lambda)\tau]$.

Let us study some properties of the solutions of (1).

Lemma 1. If the parameters τ, φ^* satisfy the inequality

$$c(A\tau + \varphi^*)(e^{a\tau} - 1)(e^{na_1\tau} + 1) < \varphi^* \tag{8}$$

then, when $t \geqslant 0$, the trajectory $x(t)$ will belong to the set of initial functions as long as it remains within the region Ω_0.

Proof. From the inequalities (6)-(8) it follows that $\|\psi^1(t) - x(t)\| < \varphi^*$ when $t \in [0, \tau]$. But then we also have $\|\psi^k(t) - x(t)\| < \varphi^*$ when $t \in [(k-1)\tau, k\tau]$ for $k = 2, 3, \ldots$

The remark following inequality (7) applies to any time interval $[(k-1-\lambda)\tau, (k-\lambda)\tau], \lambda \in [0,1]$, therefore the last inequality also holds for such a time interval.

Lemma 2. If the following inequalities hold together with inequality (8):

$$c(A\tau + \varphi^*)(e^{a\tau} - 1) < \rho; \quad \varphi^* \leqslant a^* \tag{9}$$

where ρ is the minimum distance between the surface $\partial\Omega^*$ and its image $\psi(t)$ after the time τ, then the solution of (1) $x(t), x(0) \in \bar{\Omega}^*$ cannot leave the region Ω_0 when $t \geqslant 0$.

Proof. From the condition $\varphi^* \leqslant a^*$ and (8) it follows that when $t \in [0, \tau]$, the trajectory $x(t)$ does not leave Ω_0 and the phase point lies within Ω^* when $t = \tau$. Therefore Lemma 1 applies when $t \in [\tau, 2\tau]$ and inequality (9) remains true. Then the trajectory $x(t)$ cannot leave Ω_0 for any $t \geqslant 0$.

Lemma 3. If the delay τ satisfies the inequality

$$\lambda < 1, \quad \lambda = c^2(e^{a\tau} - 1)^2(e^{na_1\tau} + 1) \tag{10}$$

and the trajectory $x(t)$ remains in $\bar{\Omega}_0$ also when $t \geqslant 0$, then its deviation from the reference trajectory $\psi^k(t)$ when $t \in [k\tau, (k+1)\tau]$, $k = 0, 1, 2, \ldots$ $(\psi^k(k\tau) = x(k\tau))$ does not exceed $z_k \to z_*$ as $k \to \infty$, i.e.

$$z_* < z^*, \quad z^* = D/(1-\lambda) \tag{11}$$
$$D = cA\tau(e^{a\tau} - 1)[1 + c(e^{a\tau} - 1)(e^{na_1\tau} + 1)]$$

and

$$z_k < z^* + \varphi^* \text{ for } k = 1, 2, \ldots \tag{12}$$

Proof. When $t \in [0, \tau]$, the deviation is estimated by means of the quantity z_1 equal to the right-hand side of inequality (6) when $t = \tau$. In order to obtain an analogous estimate for $t \in [\tau, 2\tau]$, it is sufficient according to (7) to replace φ^* in inequality (6) by $z_1(1 + e^{na_1\tau})$, i.e. $z_2 = D + \lambda\varphi^*$.

Continuing this process we obtain, at the k-th stage,

$$z_k = D(1+\lambda)(1+\lambda^2)(1+\lambda^4)\cdots(1+\lambda^{2^{k-3}}) + \lambda^{2^{k-2}}\varphi^*$$

By virtue of condition (10) $z_k \to z_*, z_* < \infty, k \to \infty$. (We note that z_* is independent of φ^*). Taking into account the fact that

$$(1+\lambda)(1+\lambda^2)\cdots(1+\lambda^{2^{k-3}}) = 1 + \lambda + \lambda^2 + \lambda^3 + \cdots + \lambda^{1+2+\cdots+2^{k-3}} < 1/(1-\lambda)$$

we obtain (11) and inequality (12) becomes obvious.

We will further assume that all regions Ω_v, bounded by the surface $V(x) = v$, $v \in [0, v_0]$, are convex, and $\partial\Omega_0$ is determined by the equation $V(x) = v_0$. We shall also assume that inequality (6) and the quantity ρ are specified for every such region, i.e. the functions $A = A(v)$, $a = a(v)$, $a_1 = a_1(v)$, $a_2 = a_2(v)$, $\rho = \rho(v)$ are known. Then $z_k, z_*, \lambda, D, z^*$ will also be functions of v.

Let us denote by $\lambda'(v), D'(v), z'(v)$ the expressions obtained from $\lambda(v), D(v), z^*(v)$ respectivley, after replacing $c(v)$ by $\sqrt{c'(v)}$, $c'(v) = \max c^2(v')$, $v' \leqslant v$. We see that $\lambda(v) \leqslant \lambda'(v)$, $D(v) \leqslant D'(v)$, $z^*(v) \leqslant z'(v)$ and the right-hand sides of these inequalities decrease monotonically as $v \to 0$, and $z'(v) \to 0$.

Theorem. In order for Ω^* to belong to the domain of attraction of the unperturbed motion $x = 0$ (1), it is sufficient that the parameters τ and φ^* satisfy the inequalities (8, 9) as well as the inequalities

$$\lambda'(v) < 1 \tag{13}$$

$$z'(v) < \rho(v), \quad 0 < v \leqslant v_0 \tag{14}$$

Proof. When conditions (8), (9) hold, then according to Lemma 2 any trajectory $x(t)$ of system (1) (with the initial function belonging to the class chosen previously) remains in Ω_0 when $t \geqslant 0$.

Since $\lambda'(v)$ decreases monotonically as $v \to 0$, condition (13) guarantees that the function $z'(v)$ exists and is continuous in $[0, v_0]$. Thus inequality (14) makes sense. Let us assume that it holds. Then we have, in particular, $z'(v_0) > \rho(v_0)$, and according to Lemma 3, when the number $k = k_1$ becomes sufficiently large, $z_k < z^* \leqslant z'(v_0)$ will hold. Therefore, beginning from at least this number and by virtue of the finite difference $\rho(v_0) - z_{k_1}$, the trajectory $x(t)$ must remain in the region $\Omega_{v^{(1)}}$, $v^{(1)} < v_0$.

Applying the previous arguments to this region, we can confirm that, beginning from some number $k = k_2 > k_1$, the trajectory $x(t)$ will remain within the region $\Omega_{v^{(2)}}$, $v^{(2)} < v^{(1)}$.

The sequence $v^{(1)}, v^{(2)}, \ldots, v^{(l)}, \ldots$ tends to zero as $l \to \infty$, since the assumption that $v^{(l)} \to v_* > 0$ is confirmed by the existence of a number l for which the difference $\rho(v_*) - z_{kl} > 0$. (We have not used the preliminary inequality $\tau < t^*$ in the proof, and it can therefore be neglected.)

Corollary 1. If the solution $x = 0$ (1) is asymptotically stable in the large when $\tau = 0$, the regions Ω_v are convex and the inequalities (13), (14), hold when $v \in (0, \infty)$, $\tau = \tau^*$ (the inequalities (8), (9) are omitted), then it is clear that the solution remains asmyptotically stable in the large when $\tau = \tau^*$, $\varphi^* < \lim z'(v)$, $v \to \infty$.

Note 1. We can use, as the minorant for $\rho(v)$, the minimum distance between the surfaces $V(x) = v$ and $V(x) = v_\tau$ where v_τ is defined as the value of the solution (or as a minorant of the solution) of the differential equation

$$v^{\cdot} = -w(v), \quad w(v) \leqslant \min(-V^{\cdot}(x))_{\tau=0}, \quad x \in \partial\Omega_v \tag{15}$$

for $t = \tau$ and initial condition $v(0) = v$.

Corollary 2. If system (1) is linear with constant coefficients and the solution $x = 0$ is asymptotically stable, then for any φ^* there exists $\tau^* > 0$, such that when $\tau \leqslant \tau^*$, the solution $x = 0$ remains asymptotically stable in the large.

Proof. According to the well-known Lyapunov theorem a positive definite quadratic form $V(x)$ such that $V^{\cdot} = U$ can be constructed for the function $U = -\|x\|^2$.

We have the estimate $\|x\|^2 \geqslant V(x)/\Lambda$ where Λ is the largest eigenvalue of the matrix of the form $V(x)$. Therefore, we obtain the estimate $\|x\|^2 \geqslant v/\Lambda$ for $x \in \partial\Omega_v$ and we can take the right-hand side of the latter inequality as $w(v)$. Thus Eq.(15) here takes the form $v^{\cdot} = -v/\Lambda$. Then

$$v_\tau = ve^{-\tau/\Lambda} \quad \text{and} \quad \rho(v) = \frac{1}{2}\Lambda^{-3/2}\tau\sqrt{v} + o(\tau).$$

We further note that (by virtue of the linear character of (1)) a sufficiently large number $A_1 > 0$ exists for which $A \leqslant A_1\sqrt{v}$, $0 < v$ and the coefficients $a, a_1, a_2, b, c, \lambda$ do not depend on v and $\lambda \to 0$ as $\tau \to 0$. Therefore $z'(v) = A_2(\tau)\sqrt{v}$, $A_2(\tau) = o(\tau)$.

This means that for sufficiently small τ the conditions of Corollary 1 will hold for any φ^*, and this proves the validity of the assertion.

Note 2. Eqs.(1) appear, for example, when solving the problems of the optimal stabilization of stationary motions (5), provided that we take into account the delay in transmitting the control signals. But in this case the Lyapunov function will, by virtue of (1), only have a time derivative with negative terms when $\tau = 0$, and for this reason the result obtained here cannot be used directly. In many cases however, the function can be rearranged into a function with a negative definite derivative /6/.

Note 3. If inequality (14) is violated when $v = v_* < v^*$, this will clearly mean that the trajectories of (1) originating in $\bar{\Omega}^*$ will arrive, after a finite time, at the region Ω_{v_*} and will remain in this region as $t \to \infty$, i.e. Ω_{v_*} is an attractor for (1).

Example. 1. When $\tau = 0$, the position of equilibrium $x = 0$ is asymptotically stable in the large for the system $x_1^{\cdot} = x_2$, $x_2^{\cdot} = -x_1 - x_2(t - \tau)$, and $V(x) = x_1^2 + x_2^2 + x_1x_2$, $V^{\cdot}(x) = -V(x)$. Let us estimate τ, φ^* for any region Ω_0 which represents the interior of the ellipse $x_1^2 + x_2^2 + x_1x_2 = v_0$.

Here

$$A = \sqrt{2v}, \; a_1 = a_2 = 1, \; K_1 = \sqrt{8}, \; K_2 = 2, \; K_3 = 1, \; a = 4,8$$

$$w(v) = v, \quad v_\tau = ve^{-\tau}, \quad \rho(v) = \sqrt{\frac{2}{3}}(1 - e^{-1/2\tau})\sqrt{v}$$

and when $\tau = 0{,}1$, $z'(v) = 0.041\sqrt{v}$, $\lambda' < 1$, $\rho(v) = 0.040\sqrt{v}$, i.e. the solution $x = 0$ remains asymptotically stable in the large for any φ^*.

Note that the asymptotic stability of linear systems with a delay and with constant coefficients can be studied using the method of *D*-decompositions /7/. By testing various values of τ we can determine, using this method, the admissible delay with a greater accuracy, but the method does not produce, for given φ^*, an estimate of the region in which the solution must always remain.

Example 2. The following expressions /8/:

$$V = \frac{x^2}{1 + x^2} + y^2, \quad V^{\cdot} = -4\left[\frac{x^2}{(1 + x^2)^4} + \frac{y^2}{(1 + x^2)^2}\right]$$

are known for the system

$$x^{\cdot} = \frac{-2x(t - \tau)}{(1 + x^2)^2} + 2y, \quad y^{\cdot} = \frac{-2(x + y)}{(1 + x^2)^2}$$

when $\tau = 0$. Here the domain of attraction of the zero order solution is the region bounded by the surface $x^2/(1 + x^2) + y^2 = v_0$, $v_0 < 1$.

When $v \leqslant v_0 = 0.5$, the regions Ω_v are convex. Here Eq.(15) has the form $v^{\cdot} = -4v(1 - v)^3$ and the quantity v_τ is given by the expression $v_\tau = 4\tau + v + \ln(1 - v) - \ln v + \ln v_\tau - \ln(1 - v_\tau)$.

Seeking the solution of the last equation in the form of the series

$$v_\tau = v + g_1(v)\tau + g_2(v)\tau^2 + \ldots$$

we obtain, up to terms of order $o(\tau)$,

$$v_\tau = v\xi, \quad \rho(v) = (1 - \sqrt{\xi})\sqrt{v}, \quad \xi = 1 - 4\frac{1-v}{1-v+v^2}\tau$$

Having found $a_1 = 2$, $a_2 = 1$, $K_1 = 2\sqrt{8}$, $K_2 = 2$, $K_3 = 2$, $A = \sqrt{8v}$, we can confirm that all the conditions of the thoerem hold, e.g. when $\tau = 0.05$, $\varphi^* = 0.1$, and the region Ω^* is determined by the parameter $v^* = 0.37$.

Example 3. When $\tau = 0$, the position of equilibrium $x = 0$ of the system

$$x_1^{\cdot} = -b_1x_1^2x_1(t-\tau) + b_2x_2, \quad x_2^{\cdot} = -b_3x_1 - b_4x_2^3, \quad (b_1 > 0, \ldots, b_4 > 0) \tag{16}$$

is asymptotically stable in the large /8/ and

$$V = b_3x_1^2 + b_2x_2^2, \quad V^{\cdot} = -2b_1b_3x_1^4 - 2b_2b_4x_2^4$$

The regions Ω_v are convex for any $v > 0$.

Taking into account the symmetry of the surfaces $V(x) = v$, we obtain

$$w(v) = gv^2, \quad v_\tau = \frac{v}{1 + g\tau v}, \quad g = \frac{2b_1b_2}{b_1b_2 + b_3b_4}$$

$$\rho(v) = \sqrt{v}\,(1 - 1/\sqrt{1 + g\tau v})/\sqrt{b_*}, \quad b_* = \max\{b_1, b_3\}$$

$$a_1 = \max\{3b_1, x_1^2, b_2, b_3, 3b_4x_2^2\}, \; a_2 = 1, \; K_1 = \sqrt{8}a_1 \; K_2 = 2, \; K_3 = b_1x_{10},$$

$$a = \sqrt{8}a_1 + b, \; b = 2b_1x_{10}, \; x_{10} = \max x^2_1, \; x \in \Omega_v.$$

It can be shown that all conditions of the theorem will hold, at least when τ, v_0 and $\varphi^* = O(\tau\sqrt{v_0})$ are sufficiently small.

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Translated by L.K.